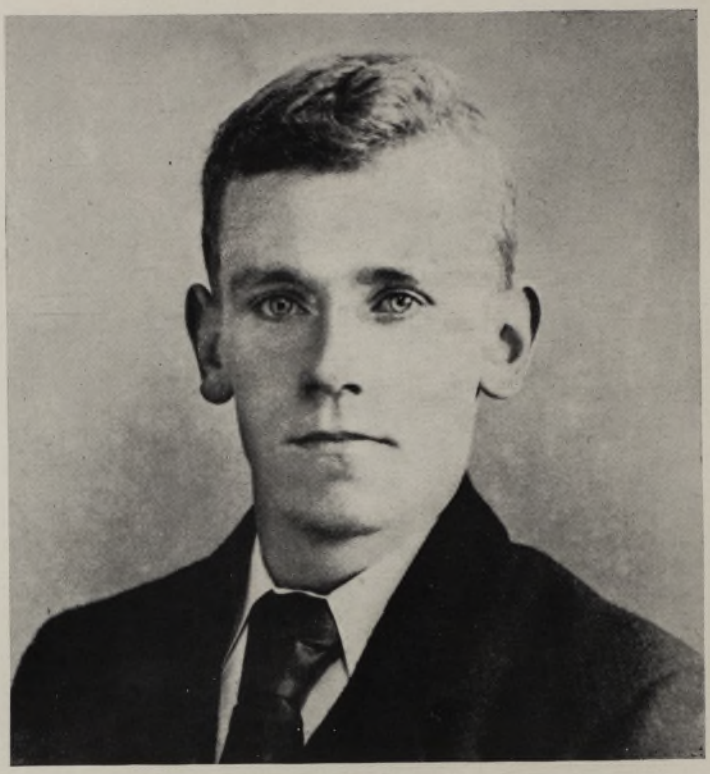

J. H. Grace

JOHN HILTON GRACE

1873-1958

(F.R.S. 1908-1922)

John Hilton Grace was born on 21 May 1873 at Halewood, a village in South Lancashire not far from Liverpool, the eldest of six children of a farmer. He received his early education at the village school, from which he went on to the Liverpool Institute. Here he came under the influence of an outstanding teacher of mathematics, J. A. Owen (who lived to the advanced age of 103), and in 1892 he entered Peterhouse, Cambridge, with an open Entrance Scholarship. His was a vintage year for Cambridge mathematics; among those who took the Mathematical Tripos in 1895 were six future Fellows of the Royal Society; besides Grace himself these were T. J. I'A. Bromwich, F. W. Carter, B. Hopkinson, E. T. Whittaker (later Sir Edmund Whittaker), and A. Young. Young was to become closely associated with Grace in his later mathematical work. There was intense rivalry, not only between the candidates but between their colleges, for the Senior Wranglership. Grace, Bromwich, and Whittaker were the most fancied candidates, and the element of doubt in Grace's case was not due to any misgivings regarding his mathematical ability, but because it was thought that his rather unstable way of life might result in his not doing himself justice. At any rate a wag pinned on the door of one of the favourites, who was known to be of a religious turn of mind, the verse

'Work, as if on that alone
Hung the issue of the day,
Pray that Grace may be sent down,
Work and pray!'

In the event, Bromwich was Senior Wrangler, and Grace and Whittaker were bracketed second. Rumour had it that Grace was in the lead at the end of the penultimate paper, but celebrated a day too soon. Whether this is true or not it is in keeping with the fact that Grace was his own worst enemy, and throughout his life he missed opportunities of achieving the greatness of which he was capable through his intemperate habits.

Grace became a fellow of Peterhouse in 1897, and was later a college lecturer at Peterhouse and Pembroke, and held one of the two University lectureships in mathematics then existing. He served on the Council of the London Mathematical Society from 1903 to 1916, being Secretary from 1904 to 1915, and subsequently a Vice-President. He was elected a Fellow of the Royal Society in 1908. He spent the year 1916-1917 as Visiting

Professor at Lahore, and from 1917 to 1919 he deputized for Professor H. M. Macdonald at Aberdeen, in the latter's absence on war service. He then returned to Cambridge, but in 1922 a breakdown caused his permanent withdrawal from academic life. For some years he lived in Norfolk, but later returned to Cambridge, where he lived very quietly until 1956. His last months were spent in Huntingdon, where he died on 4 March 1958.

An examination of Grace's published work reveals some unusual features. In ten papers published between 1898 and 1904 he reveals the topics which were to remain a permanent interest in his life. Then, at a time when a mathematician might be expected to be most productive, there is a gap of a dozen years, followed in 1916-1918 by half a dozen papers. These years, significantly, are these of his absence from Cambridge. The conclusion is irresistible that his teaching activities during the years he was at Cambridge absorbed most of his energies. He threw himself heart and soul into his teaching, and several of his pupils of that time have paid tribute to its outstanding qualities; he demanded the best of his students, and those whose interests lay in fields remote from his own found him difficult to keep up with. But to those whose interests lay close to his own he was immensely stimulating and suggestive.

After 1918 there is another gap in the record of publications, clearly due to the impending breakdown in health. But, from his retirement, he produced a steady flow of papers from 1926 to 1930, and two later papers, the work of his old age.

Grace was fundamentally a geometer; though many of his papers deal with invariant theory his outlook on this subject was much more geometrical than that of most algebraists of his period. His most important work was his collaboration with Young in the book *Algebra of invariants*, published in 1903, which first presented the work of Clebsch and Gordan to British mathematicians; an outstanding exposition of the subject. Many of his papers deal with various aspects of this topic, and they are characterized by great ingenuity and elegance. A fine specimen of his early work is his proof (1902), in the paper 'The zeros of a polynomial', that if A and B are points in the Argand diagram, representing two roots of the equation $f(z) = 0$, where $f(z)$ is a polynomial of degree n, then the derived polynomial $f'(z)$ has at least one zero inside the circle whose centre is the mid-point of AB and whose radius is $\frac{1}{2}AB \cot \pi/n$. The key to this result is the following lemma, for which he gives a simple and entirely characteristic proof: if $f(z)$, considered as a homogeneous binary form, is apolar to $\phi(z)$, then it has a zero in any circle enclosing all the zeros of $\phi(z)$. The idea of apolarity seems to have intrigued Grace, and he handled it in several papers with great skill, for instance, in his determination of the cases in which binary and ternary forms can be constructed with prescribed polar systems (1928), or in his proof that a ternary n–ic can be expressed as a symmetric determinant whose elements are linear forms (1927), or, again, in his very elegant generalization of Stroh's lemma to an arbitrary number of variables (1928). There is an epi-

grammatic flavour about most of his papers: generally a simple but striking and unexpected idea is introduced, which leads directly to the desired result. His last paper (1947), occupying barely a page, disposes of the possibility of an extension of Aronhold's theorem (that a plane quartic curve is uniquely determined by a suitable set of seven bitangents) to the sextic curve of intersection of a quadric and a cubic surface, by a simple remark, of which the referee observed that the idea was worthy of its author. Of interest, also, is his proof (1930) that a basis for the ideal formed by the polynomials in the coefficients of a binary form, which vanish when the form has some assigned projective property, is the set of coefficients of a number of covariants of the form.

The other main group of Grace's papers is more frankly geometrical. In his first, and only long, paper (1898) he exploits systematically the representation of lines and linear complexes by points and hyperspheres in four dimensions, and obtains very neat proofs of a number of 'chain' theorems for circles and spheres. It is from one of these theorems, incidentally, that he deduces the curious theorem on the 'extension of the double six'. If five lines in space have a common transversal, then any four of the five have a second transversal, and the theorem of the double six states that the five lines so obtained are themselves met by another line. Grace shows that if six lines have a common transversal, then the same is true of the six lines derived, by the double six theorem, from the six sets of five lines which may be selected from the given set of six. In another paper (1929) he shows, by ingenious use of invariantive arguments, that the problem of constructing a rational normal curve of order $2r+1$ with four assigned osculating r-spaces is poristic, while that of constructing a rational normal curve of order r^2+4r+1 with $r+4$ assigned osculating r-spaces admits in general a unique solution.

Three isolated papers deal with the theory of numbers, two of these (1918) concern problems on Diophantine approximation and the third (1927) contains a simple proof of Lagrange's theorem, that any positive integer can be expressed as a sum of four integral squares, by an ingenious use of lattices. Mention should also be made of an elementary textbook on analytic geometry (published jointly with F. Rosenberg) well in advance of its time in its use of general methods, and his articles on line geometry in the tenth and eleventh editions of the *Encyclopaedia Britannica*.

Many of Grace's papers seem, at a time when general theories are the current mathematical fashion, to deal with very special and particular problems. Grace was, in fact, an amateur in the true sense of the word; he tackled a problem simply because he found it interesting. None of his papers are trivial, even when they deal with elementary topics. Most of them have a deceptive appearance of simplicity, since they depend on the exploitation of a quite simple idea. But to get these ideas, more than once or twice in a mathematical lifetime, one must have at least something of Grace's remarkable perception, which seizes on the really significant points in a

problem, and his instinctive feeling for what is, and what is not, really relevant. His uncanny facility is something not easily imitated.

As a man he was sociable and friendly, with a whimsical attitude to life, and a vigorous humour. His range of interests was wide; as a young man he was a keen rock-climber, and had a great love and expert knowledge of the Turf. His premature retirement removed a colourful figure from Cambridge life, and left behind only the memory of a very gifted and stimulating man.

It is not possible to mention by name all the numerous friends and pupils of Grace who have very kindly given me their impressions of him in his prime. I am, however, particularly indebted, for family details, to his nephew, Dr Alan Robertson, and to Professor H. W. Turnbull, F.R.S., and Dr W. L. Edge for sending me letters written by Grace.

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